NAME:

Statistics 111 Summer Session II

Homework Three

You are allowed to discuss problems with other students, but the final answers must be your own work.

For all problems that require calculation, YOU MUST ATTACH SEPARATE PAGES, NEATLY WRITTEN, THAT SHOW YOUR WORK.

Please mark your answer in the space provided. As a general rule, each blank counts for one point unless otherwise specified. If necessary work is not shown, or if that work is substantially wrong, then you will not get credit even if the answer is correct. (The obvious purpose of this is to prevent students from mindlessly copying each other's answers.)

Report all numerical answers to at least two correct decimal places.

DUE DATE: START of class on Tuesday, July 17, 2018.

1. (**Normal Distributions**) Heights of 10 year olds, regardless of gender, closely follow a normal distribution with mean 55 inches and standard deviation 6 inches.

   (a) What is the probability that a randomly chosen 10 year old is between 60 and 65 inches?

   (b) If the tallest 10% of the class is considered "very tall", what is the height cutoff for "very tall"?

   (c) The minimum height requirement for *Batman the Ride* at Six Flags Magic Mountain is 54 inches. What percent of 10 year olds cannot go on this ride?

2. (**Bivariate Distribution—Discrete**) Consider the joint probability mass function for bivariate discrete random variables defined by the following table:

	x=1	x=2	x=3
y=0	0	0.09	0.15
y=4	0.15	0.04	0.25
y=5	0.07	0.05	c

(a) What is the value of c?

(b) What is $E(X|y = 4)$?

(c) Are X and Y independent?

(d) What is the correlation between X and Y?

3. (**Bivariate Distribution—Continuous**) Suppose $f(x,y) = 6x$, for $x + y \leq 1$ with both x and y restricted to be between 0 and 1. (Hint: when working with bivariate densities, it is always a good idea to draw the support.)

   (a) What is the expected value of X?

   (b) Are X and Y independent?

   (c) What is the correlation between X and Y?

4. (**Functions of a Random Variable**) Suppose that the probability density function of X follows:

$$f(x) = \begin{cases} e^{-x} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$$

Determine the p.d.f. of $Y = X^{1/2}$

5. (**Linear Combinations**) Suppose that an automobile dealer pays an amount X (in thousands of dollars) for a used car and then sells it for an amount Y. Suppose that the random variables X and Y have the following joint p.d.f:

$$f(x,y) = \begin{cases} \frac{1}{36}x & \text{for } 0 < x < y < 6 \\ 0 & \text{otherwise} \end{cases}$$

Determine the dealer's expected gain from the sale. (2 points)

6. (**Central Limit Theorem**) Suppose that people attending a party pour drinks from a bottle containing 63 ounces of a certain liquid. Suppose also that the expected size of each drink is 2 ounces, that the standard deviation of each drink is 1/2 ounce, and that all drinks are poured independently. Determine the probability that the bottle will not be empty after 36 drinks have been poured. (2 points)

How hard was this homework assignment compared to the last one and how many hours did it take you to finish it? Are you feeling better about the class? As you may have noticed, I've been making some changes to incorporate your suggestions from the last homework. Is there anything else you still want us to do differently in class?